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Effect of rotation on the stability of a doubly diffusive fluid layer in a porous medium

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Abstract—The convective stability of a doubly diffusive fluid saturating a rotating porous medium is considered. It is observed that (i) a rotating layer can be destabilized by a bottom-heavy arrangement; (ii) in general, rotation stabilizes the system though under certain conditions it destabilizes the system, (iii) rotation has little effect on the stability of the fluid layer under certain conditions, (iv) under some conditions three Rayleigh numbers are required to specify linear stability criteria.

INTRODUCTION

EFFECTS of rotation on doubly diffusive fluid systems find application in various branches of modern science like biochemistry, oceanography, stellar convection, etc. Pearlstein [1] has given a brief survey of important findings of various workers in these fields. It is observed that rotation in general enhances the convective stability of a doubly diffusive fluid, but for certain Darcy–Taylor numbers, Darcy–Prandtl numbers and Darcy–Schmidt numbers it destabilizes such a fluid. Another branch of science in which rotation plays an important role is geophysics. It is known that the earth's crust consists of a mixture of different types of fluids like oil, water, gases, etc. the temperature of which increases as one goes deep inside. Also, constant angular velocity of the earth about its geographical axis gives rise to Coriolis force. Hence any attempt to study convective currents in geothermal systems will lead to the problem of finding the effect of rotation on the stability of a multicomponent fluid the components of which can diffuse relative to one another. Since diffusivity of mass is less than diffusivity of heat under certain conditions, oscillatory convective motions may set in. As the earth's crust is essentially a porous medium, its porosity may also affect the stability of the system. Stability of a non-rotating doubly diffusive fluid saturating a porous medium has been studied by Nield [2], Rudraiah *et al.* [3], Taunton *et al.* [4], Wankat and Schowalter [5], Rudraiah and Prabhamani [6], Patil and Rudraiah [7], and Patil and Vaidyanathan [8]. Pearlstein [1] has given a detailed analysis for a rotating doubly diffusive Newtonian fluid layer.

The aim of this paper is to study the effect of rotation on the convective stability of a doubly diffusive fluid saturating a porous medium. It has been assumed that isotropy of the medium is not disrupted due to rotation.

MATHEMATICAL FORMULATION

Consider a quiescent layer of a Boussinesq doubly diffusive fluid layer saturating a porous medium extending to infinity in the x'- and y'-directions and bounded by free horizontal boundaries $z' = 0$ and L which are maintained at temperatures T_0 and T_L, respectively. Let the system be rotating with an angular velocity $\mathbf{\Omega}$ about the z'-axis. The concentration of the diffusing component is held at C_0 and C_L at $z' = 0$ and L, respectively.

The governing equations are

$$\nabla \cdot \mathbf{q} = 0 \quad (1)$$

$$\frac{1}{\phi}\left[\frac{\partial \mathbf{q}}{\partial t'} + \mathbf{q}|\mathbf{q}| + 2(\mathbf{\Omega} \times \mathbf{q})\right] = -\nabla \bar{p} - \frac{\mu}{\bar{\rho} k}\mathbf{q} + \frac{\rho \mathbf{g}}{\bar{\rho}} \dots \quad (2)$$

$$\frac{\partial T}{\partial t'} + M\mathbf{q} \cdot \nabla T = \kappa \nabla^2 T \quad (3)$$

$$\phi \frac{\partial C}{\partial t'} + \mathbf{q} \cdot \nabla C = D \nabla^2 C. \quad (4)$$

Equation (2) is the well-known Darcy–Oberbeck–Boussinesq (DOB) equation [9] modified for a rotating system. It is assumed that rotation does not disrupt the isotropy of the medium. The Darcy model is valid for $k/L^2 < 10^{-3}$ [10]. Equation (4) is that of Poulikakos [11] and Gray [12]. The equation of state is taken in the form

$$\rho = \bar{\rho}[1 - \alpha_T(T - \bar{T}) + s\alpha_c(C - \bar{C})]$$

where $s = 1$ (-1) if the partial molar density of the

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NOMENCLATURE

- a horizontal wave number, $(a_x^2+a_y^2)^{1/2}$
- a_x wave number in the x-direction
- a_y wave number in the y-direction
- a_c critical wave number
- c_p heat capacity
- C solute concentration
- C_b basic state solute concentration
- C_0 solute concentration at $z' = 0$
- C_L solute concentration at $z' = L$
- $\bar{C}$ mean of C_0 and C_L, $(C_0+C_L)/2$
- C' perturbation in solute concentration
- D effective diffusivity
- $\mathbf{g}$ acceleration due to gravity, $(0, 0, -g)$
- $\hat{i}, \hat{j}, \hat{k}$ unit vectors along the x'-, y'-, z'-directions
- k permeability of the medium
- L thickness of the porous layer
- M $(\bar{\rho}c_p)_f/(\bar{\rho}c_p)_m$
- p pressure
- p_b basic state pressure
- $\tilde{p}$ $p/\bar{\rho}-1/2|(\Omega/\phi)\times \mathbf{r}|^2$
- $\tilde{p}_b$ $p_b/\bar{\rho}-1/2|(\Omega/\phi)\times \mathbf{r}|^2$
- Pr Prandtl number, ν/κ
- p' perturbation in $\tilde{p}$
- $\mathbf{q}$ seepage velocity vector
- $\mathbf{q}_b$ basic state seepage velocity, $(0, 0, 0)$
- $\mathbf{q}'$ perturbation in seepage velocity, (u', v', w')
- $\mathbf{r}$ position vector
- R_{tcrit} critical thermal Rayleigh number
- R_t thermal Rayleigh number, $\alpha_T(T_0-T_L)Lkg\bar{\rho}/\kappa\mu$
- R_c salinity Rayleigh number, $\alpha_c(C_L-C_0)Lkg\bar{\rho}/D\mu$
- s constant which takes the values 1 or -1
- S dimensionless concentration, $C'\phi/(C_L-C_0)$
- Sc Schmidt number, ν/D
- t dimensionless time, $(\mu\phi/\bar{\rho}k)t'$
- t' time
- T temperature
- Ta Darcy–Taylor number, $(2\Omega k/\phi\nu)^2$
- T_b basic state temperature
- T_0 temperature at $z' = 0$
- T_L temperature at $z' = L$
- $\bar{T}$ mean of T_0 and $T_{L'}$, $(T_0+T_L)/2$
- T' perturbation in temperature
- W dimensionless velocity in the z'-direction, $(\bar{\rho}k/\mu\phi L)w'$
- x', y', z' Cartesian coordinates
- x, y, z dimensionless coordinates, x'/L, y'/L, z'/L.

Greek symbols

- α $(n^2\pi^2+a^2)^{1/2}$
- α_c coefficient of solute mass expansion
- α_T coefficient of volume expansion
- ζ dimensionless vorticity in the z'-direction, $(\bar{\rho}k/\mu\phi)\zeta'$
- ζ' z'-component of vorticity
- θ dimensionless temperature, $T'/(T_0-T_L)$
- κ thermal diffusivity, $\lambda_m/(\bar{\rho}c_p)_m$
- λ_m thermal conductivity, $\phi\lambda_f+(1-\phi)\lambda_s$
- Λ_1 Darcy–Prandtl number, $Pr\,(L^2\phi/k)$
- Λ_2 Darcy–Schmidt number, $Sc\,(L^2\phi^2/k)$
- μ coefficient of viscosity
- ν kinematic viscosity, $\mu/\bar{\rho}$
- ρ density of the doubly diffusive fluid
- ρ_b basic state density
- $\bar{\rho}$ density of the fluid at $T = \bar{T}$, $C = \bar{C}$
- ρ' perturbation in density
- ϕ porosity
- Ω angular velocity vector, $(0, 0, \Omega)$.

Other symbols

- ∇ $(\partial/\partial x')\hat{i}+(\partial/\partial y')\hat{j}+(\partial/\partial z')\hat{k}$
- ∇^2 $\partial^2/\partial x'^2+\partial^2/\partial y'^2+\partial^2/\partial z'^2$
- ∇_1^2 $\partial^2/\partial x'^2+\partial^2/\partial y'^2$.

Subscripts

- f fluid
- m solid–fluid mixture
- s solid.

diffusing component is greater (less) than that of the solvent.

Consider the basic state in which

$$\left.\begin{aligned} &\mathbf{q} = \mathbf{q}_b = (0, 0, 0) \\ &\tilde{p} = \tilde{p}_b, \quad T = T_b, \quad C = C_b, \quad \rho = \rho_b \end{aligned}\right\} \quad (5)$$

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial y'} = \frac{\partial}{\partial t'} = 0, \quad \frac{\partial}{\partial z'} \neq 0$$

where

$$\left.\begin{aligned} &\frac{\partial \tilde{p}_b}{\partial z'} = \frac{-\rho_b}{\bar{\rho}}g = -g[1-\alpha_T(T_b-\bar{T})+s\alpha_c(C_b-\bar{C})] \\ &\frac{\partial T_b}{\partial z'} = \frac{T_L-T_0}{L}, \quad \frac{\partial C_b}{\partial z'} = \frac{C_L-C_0}{L}. \end{aligned}\right\} \quad (6)$$

On the basic state given by equations (5) impose a small perturbation

$$\left.\begin{array}{l}\mathbf{q}=\mathbf{q}', \quad \tilde{p}=\tilde{p}_b(z')+p', \quad T=T_b(z')+T' \\ C=C_b(z')+C', \quad \rho=\rho_b(z')+\rho'\end{array}\right\} \quad (7)$$

where primed quantities denote the perturbations. Linearized dimensionless equations for the perturbations are

$$\left[\frac{\partial}{\partial t}-\frac{\nabla^2}{\Lambda_1}\right]\theta = MW \quad (8)$$

$$\left[\frac{\partial}{\partial t}-\frac{\nabla^2}{\Lambda_2}\right]S = -W \quad (9)$$

$$-\left[\frac{\partial}{\partial t}+1\right]\nabla^2 W = Ta^{1/2}\frac{\partial\zeta}{\partial z}+\frac{sR_c}{\Lambda_2}\nabla_1^2 S-\frac{R_t}{\Lambda_1}\nabla_1^2\theta \ldots \quad (10)$$

$$\left[\frac{\partial}{\partial t}+1\right]\zeta = Ta^{1/2}\frac{\partial W}{\partial z}. \quad (11)$$

Boundary conditions imposed at perfectly conducting permeable, stress free boundaries at $z = 0$ and 1 are

$$W=\frac{\partial\zeta}{\partial z}=\theta=S=0. \quad (12)$$

Following Chandrasekhar [13] the perturbations are assumed to be

$$[W,\zeta,\theta,S]=[W_0(z),\zeta_0(z),\theta_0(z),S_0(z)] \times\exp(\mathrm{i}a_x x+\mathrm{i}a_y y+\sigma t). \quad (13)$$

Substituting equation (13) in equations (8)–(11) one obtains

$$\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_1}\right]\theta_0 = MW_0 \quad (14)$$

$$\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_2}\right]S_0 = -W_0 \quad (15)$$

$$-(\sigma+1)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)W_0 = Ta^{1/2}\frac{\mathrm{d}\zeta_0}{\mathrm{d}z} -a^2\left(\frac{sR_c}{\Lambda_2}S_0-\frac{R_t}{\Lambda_1}\theta_0\right)\ldots \quad (16)$$

$$(\sigma+1)\zeta_0 = Ta^{1/2}\frac{\mathrm{d}W_0}{\mathrm{d}z}. \quad (17)$$

The corresponding boundary conditions are

$$W_0=\frac{\mathrm{d}\zeta_0}{\mathrm{d}z}=\theta_0=S_0=0 \quad \text{at} \quad z=0,1. \quad (18)$$

Equations (14)–(17) could be reduced to the following equation in W_0:

$$-(\sigma+1)^2\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_2}\right]\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_1}\right] \times\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right]W_0 = Ta\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_2}\right] \times\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_1}\right]\frac{\mathrm{d}^2W_0}{\mathrm{d}z^2}+(\sigma+1)a^2\frac{sR_c}{\Lambda_2} \times\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_1}\right]W_0+(\sigma+1)a^2\frac{MR_t}{\Lambda_1} \times\left[\sigma-\frac{\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-a^2\right)}{\Lambda_2}\right]W_0. \quad (19)$$

That all even derivatives of W_0 vanish at $z = 0$ and 1 can be verified from equations (14) to (17) and boundary conditions (18), which are satisfied by

$$W_0(z) = W_n \sin n\pi z. \quad (20)$$

Substitution of equation (20) in equation (19) leads to

$$[\alpha^2(\sigma+1)^2+n^2\pi^2\,Ta][\alpha^2+\sigma\Lambda_2][\alpha^2+\sigma\Lambda_1] -a^2(\sigma+1)[sR_c(\alpha^2+\sigma\Lambda_1)+MR_t(\alpha^2+\sigma\Lambda_2)]=0\ldots \quad (21)$$

Equation (21) is a fourth-degree equation in σ which can be rearranged as

$$\sigma^4+B_1\sigma^3+C_1\sigma^2+D_1\sigma+E_1=0 \quad (22)$$

where

$$B_1=\alpha^2\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)+2$$

$$C_1=\frac{\alpha^4}{\Lambda_1\Lambda_2}+2\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)\alpha^2+1 +\frac{1}{\alpha^2}\left[\pi^2n^2\,Ta-a^2\left(\frac{sR_c}{\Lambda_2}+\frac{MR_t}{\Lambda_1}\right)\right]$$

$$D_1=\frac{2\alpha^4}{\Lambda_1\Lambda_2}+\alpha^2\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)-\frac{a^2}{\Lambda_1\Lambda_2}(sR_c+MR_t) +\pi^2n^2\,Ta\left(\frac{1}{\Lambda_1}+\frac{1}{\Lambda_2}\right)-\frac{a^2}{\alpha^2}\left(\frac{sR_c}{\Lambda_2}+\frac{MR_t}{\Lambda_1}\right)$$

$$E_1=\frac{\alpha^4}{\Lambda_1\Lambda_2}+\frac{\pi^2n^2\,Ta}{\Lambda_1\Lambda_2}\alpha^2-\frac{(sR_c+MR_t)}{\Lambda_1\Lambda_2}a^2. \quad (23)$$

For the marginal state of instability via stationary convection (when the principle of exchange of stabilities is valid), $\sigma = 0$ which gives, from $E_1 = 0$, the following condition on R_c and R_t:

$$\alpha^4 + \pi^2 n^2\, Ta\, \alpha^2 - (sR_c + MR_t)a^2 = 0.$$

For the lowest mode $n = 1$

$$MR_t = -sR_c + \frac{(\pi^2+a^2)^2}{a^2} + \pi^2\, Ta \frac{(\pi^2+a^2)}{a^2}. \quad (24)$$

The critical wave number and thermal Rayleigh number are given by

$$a_c^2 = \pi^2\sqrt{(1+Ta)} \quad (25)$$

$$MR_{tcrit} = -sR_c + \pi^2[1+\sqrt{(1+Ta)}]^2. \quad (26)$$

Note that R_{tcrit} is a linear function of R_c and is independent of Λ_1, Λ_2 for a given Ta.

For a non-rotating system ($Ta = 0$)

$$a_c = \pi, \quad MR_{tcrit} = -sR_c + 4\pi^2.$$

From the theory of algebraic equations, it can be verified that for $E_1 < 0$, instabilities will grow in the system. The marginal state of instability via oscillatory convection (i.e. overstability) will manifest itself only if σ is completely imaginary, say $\sigma = i\omega$. This leads to

$$(\omega^4 - C_1\omega^2 + E_1) - i\omega(B_1\omega^2 - D_1) = 0.$$

Equating real and imaginary parts to zero, the restraint on R_c, R_t is given by

$$B_1C_1D_1 - B_1^2E_1 - D_1^2 = 0.$$

Correspondingly, for $B_1C_1D_1 - B_1^2E_1 - D_1^2 < 0$ oscillatory disturbances will grow in the system. Similar observations have been made by Taunton *et al.* [4] for a non-rotating system.

For the lowest mode $n = 1$

$$MR_t = -sR_c\frac{\alpha^2+\sigma\Lambda_1}{\alpha^2+\sigma\Lambda_2} + \frac{\alpha^2(\sigma+1)(\alpha^2+\sigma\Lambda_1)}{a^2} + \frac{\pi^2\, Ta\,(\alpha^2+\sigma\Lambda_1)}{a^2(\sigma+1)}.$$

Thus R_t is a function of R_c, a, Ta, Λ_1 and Λ_2. Setting $\sigma = i\omega$ and splitting into real and imaginary parts one obtains

$$MR_t = -sR_c\left[\frac{\alpha^4+\omega^2\Lambda_1\Lambda_2}{\alpha^4+\omega^2\Lambda_2^2}\right] + \frac{\alpha^4-\omega^2\Lambda_1\alpha^2}{a^2} + \frac{\pi^2\, Ta\,(\alpha^2+\omega^2\Lambda_1)}{a^2(1+\omega^2)} + i\omega\alpha^2M^* \quad (27)$$

where

$$M^* = \left[\frac{\Lambda_2-\Lambda_1}{\alpha^4+\omega^2\Lambda_2^2}\right]sR_c + \frac{\alpha^2+\Lambda_1}{a^2} + \frac{\pi^2\, Ta\left(\frac{\Lambda_1}{\alpha^2}-1\right)}{a^2(1+\omega^2)}.$$

Since R_t must be real, either $\omega = 0$ which leads to steady neutral stability or $M^* = 0$.

OSCILLATORY NEUTRAL STABILITY

In this case $\omega \neq 0$, hence $M^* = 0$, i.e. ω must satisfy

$$\delta\omega^4 + \beta\omega^2 + \gamma = 0 \quad (28)$$

where

$$\delta = \Lambda_2^2(\alpha^2+\Lambda_1) \quad (29)$$

$$\beta = (\alpha^2+\Lambda_1)(\alpha^4+\Lambda_2^2) + \Lambda_2^2\pi^2\, Ta\left(\frac{\Lambda_1}{\alpha^2}-1\right) - a^2sR_c(\Lambda_1-\Lambda_2)$$

$$\gamma = \alpha^4(\alpha^2+\Lambda_1) + \pi^2\, Ta\,\alpha^4\left(\frac{\Lambda_1}{\alpha^2}-1\right) - a^2sR_c(\Lambda_1-\Lambda_2).$$

Equation (28) is a quadratic equation in ω^2 which can have more than one positive root for fixed 'a', R_c, Ta, Λ_1 and Λ_2, when multiple oscillatory neutral solutions can exist. For the non-rotating system ($Ta = 0$) the frequency is given by

$$\omega^2 = \frac{a^2sR_c(\Lambda_1-\Lambda_2)}{(\alpha^2+\Lambda_1)\Lambda_2^2} - \frac{\alpha^4}{\Lambda_2^2} \quad (30)$$

and for the rotating singly diffusive system ($\Lambda_1 = \Lambda_2$), the frequency is given by

$$\omega^2 = \frac{\pi^2\, Ta\left(1-\frac{\Lambda_1}{\alpha^2}\right)}{\alpha^2+\Lambda_1} - 1. \quad (31)$$

Thus oscillatory instabilities are possible both in the non-rotating doubly diffusive system ($Ta = 0$) and in the rotating singly diffusive system ($\Lambda_1 = \Lambda_2$) provided $\omega^2 > 0$ since frequency ω must be real.

To determine the critical thermal Rayleigh number, R_{tcrit}, the following procedure is adopted.

(i) One first checks whether equation (28) gives positive values for ω^2. If not, oscillatory instability is not possible and R_{tcrit} is given by equation (26) and a_c by equation (25).

(ii) If equation (28) gives positive values for ω^2 then the minimum over 'a' of equation (27) with ω^2 given by equation (28) is compared with R_{tcrit} given by equation (26) and the smaller value is R_{tcrit} and the corresponding critical wave number is a_c.

NUMERICAL RESULTS AND DISCUSSIONS

Convective stability of a doubly diffusive fluid saturating a rotating porous medium has been studied using linear stability analysis. DOB equations modified for a rotating system have been used to describe the flow of fluid through a porous medium. It is assumed that the isotropy of the medium is not disrupted by rotation. Numerical computations have been carried out for different values of Λ_1, Λ_2, Ta and sR_c. Throughout the analysis $M = 1$ is assumed. The results are given for different values of sR_c instead of considering $s = 1, -1$ separately, because $sR_c > 0$ implies either heavier solute is added at the top or lighter solute at the bottom and $sR_c < 0$ means heavier solute is introduced at the bottom or the lighter one at the top.

The critical thermal Rayleigh number obtained for $\Lambda_1 = 4\times10^3$, $\Lambda_2 = 4\times10^5$ are shown in Fig. 1 and Table 1. From Fig. 1, one may observe that the effect of rotation is to stabilize the fluid against convection. Also, the shape of the R_{tcrit}–sR_c plots is virtually unaffected by rotation. In the region of steady onset, where sR_c is positive for the data considered, the linearity is retained exactly according to equation (26). In the region of oscillatory onset, the deviations from linearity are very slight. The solute Rayleigh number at which the mode of instability changes also is affected very little. a_c is independent of sR_c in the region of stationary onset, while it may be seen from Table 1 that a_c varies slowly with sR_c for oscillatory onset. The last column in Table 1 indicates the mode of convection that sets in at the onset of instability for different values of sR_c. It is to be noted that oscillations are possible only for $sR_c < 0$. Figure 1 shows also that rotation inhibits both steady and oscillatory convection to very nearly the same extent. Results obtained for smaller values of Λ_1, Λ_2, where the ratio of Λ_2 to Λ_1 is considerably smaller, are shown in Fig. 2. The difference in the R_{tcrit}–sR_c plot for $\Lambda_1 = 1000$, $\Lambda_2 = 1500$ can be noted from Fig. 2. The variation with Ta of the solute Rayleigh number, at which, the mode of instability changes is more apparent in this case.

For $\Lambda_1 = 50$, $\Lambda_2 = 5000$, $Ta = 100$ it is found that the convection sets in through the oscillatory mode for all negative sR_c. The mode changes for $sR_c \simeq 593$ which are noticed from numerical results are shown in Fig. 3. It is observed from Fig. 3 that for a bottom heavy arrangement ($s = 1$, $R_c < 0$ or $s = -1$, $R_c > 0$) an increase in solute concentration causes a decrease in R_{tcrit} for a certain range of sR_c. A further increase in the solute concentration causes an increase in R_{tcrit}. In other words, the R_{tcrit}–sR_c plot is found to have a minimum. When $sR_c \to -\infty$, it is seen from numerical calculations that $R_{tcrit} \to \infty$ through the oscillatory mode (shown as inset) and when sR_c tends to infinity via positive values $R_{tcrit} \to -\infty$ through the stationary mode.

Under certain conditions it is observed that rotation has a destabilizing effect on the system while a further increase in Ta stabilizes the system. Typical graphs are shown in Fig. 4 for $\Lambda_1 = 2$, $\Lambda_2 = 5$ for different values of sR_c. The convection sets in through the oscillatory mode only for all Ta for the data under consideration. It can be seen from Fig. 4 that, when sR_c increases numerically (with $sR_c < 0$) R_{tcrit} increases. Also numerical results show that $R_{tcrit} \to \infty$ through the oscillatory mode when $sR_c \to -\infty$ and $R_{tcrit} \to -\infty$ via the stationary mode when sR_c approaches infinity through positive values (not shown in Fig. 4).

Though the general effect of rotation is to stabilize

Table 1. Values of the critical thermal Rayleigh number R_{tcrit} and the critical wave number a_c for various Darcy–Taylor numbers Ta and solute Rayleigh numbers R_c. The mode of instability depends only on sR_c in the cases shown here and appears in the last column. The results are for $\Lambda_1 = 4000$, $\Lambda_2 = 400\,000$

	Ta										
	0		10		100		500		1000		
sR_c	R_{tcrit}	a_c	R_{tcrit}	a_c	R_{tcrit}	a_c	R_{tcrit}	a_c	R_{tcrit}	a_c	Mode of instability
10^5	−99 960.5	3.14159	−99 816.1	5.72134	−98 794.9	9.95933	−94 603.6	14.8631	−89 486.1	17.6709	Stat
10^4	−9960.52	3.14159	−9816.10	5.72134	−8794.92	9.95933	−4603.64	14.8631	513.864	17.6709	Stat
10^3	−960.522	3.14159	−816.097	5.72134	205.076	9.95933	4396.36	14.8631	9513.86	17.6709	Stat
0	39.4784	3.14159	183.903	5.72134	1205.08	9.95933	5396.36	14.8631	10 513.9	17.6709	Stat
-10^3	49.8265	3.14342	195.775	5.72146	1227.08	9.96032	5457.98	14.8973	10 621.7	17.7644	Osc
-10^4	139.388	3.16087	286.146	5.72264	1319.00	9.92745	5552.62	14.8315	10 718.4	17.6804	Osc
-10^5	1034.85	3.36739	1189.85	5.73559	2237.77	9.61572	6497.22	14.2329	11 682.5	16.9242	Osc

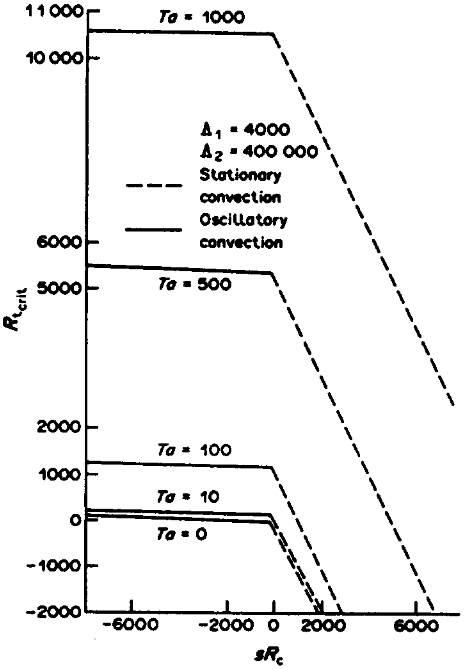


FIG. 1. Variation of R_{tcrit} with sR_c for several Darcy–Taylor numbers.

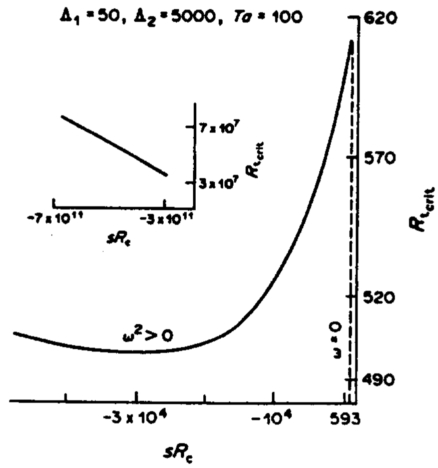


FIG. 3. R_{tcrit}–sR_c plot having a minimum. Destabilization of a bottom heavy arrangement.

the system, it is observed that for certain data rotation has no significant effect. This is clear from Table 2.

The numerical computations show that for certain values of Λ_1, Λ_2, Ta and sR_c the critical wave number a_c undergoes a sudden jump as the convection pattern changes from the stationary to the oscillatory mode (in Table 3 (S) and (O) indicate the modes of convection). Such a behaviour was observed by Chandrasekhar [13] for the convective stability of a single component rotating Newtonian fluid layer in the presence of a magnetic field.

The existence of two positive frequencies which correspond to two oscillatory neutral solutions is shown in Figs. 5(a) and (b) for $\Lambda_1 = 50$, $\Lambda_2 = 5000$. It can be seen from these figures that the oscillatory neutral stability curves are closed, but the region is connected in the topological sense and hence only one R_{tcrit} is necessary to express the stability criteria. Figure 6 shows closed disconnected neutral curves when two oscillatory modes exist for $\Lambda_1 = 5000$, $\Lambda_2 = 50$, $Ta = 100$, $sR_c = 600$. In this case $\Lambda_1 > \Lambda_2$. As can be seen from the figure, three critical thermal Rayleigh numbers are necessary to specify the regions of stability. For $R_t < -123\,372$, the system is stable. For $-123\,372 < R_t < -283$, instability sets in through

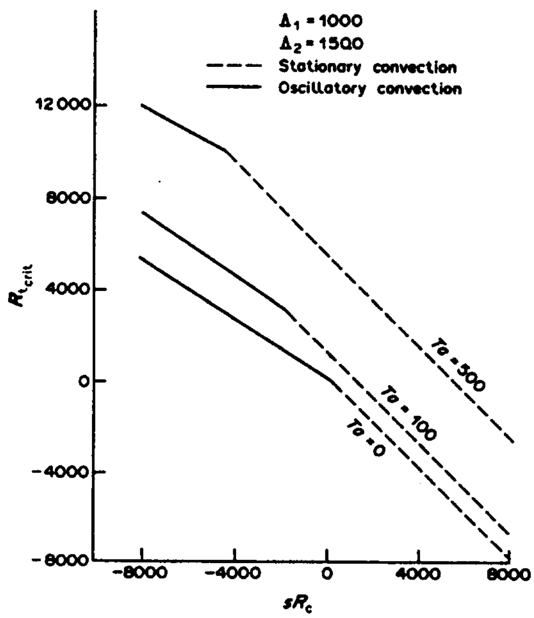


FIG. 2. R_{tcrit}–sR_c plot for several Darcy–Taylor numbers.

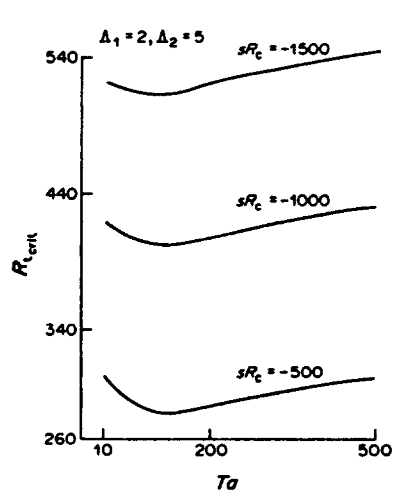


FIG. 4. R_{tcrit}–Ta plot for the oscillatory mode having a minimum. Rotation destabilizing for certain data.

Table 2. Values of R_{tcrit} for various Darcy–Taylor numbers. R_{tcrit} is affected very little by rotation for $\Lambda_1 = 5$, $\Lambda_2 = 10$ and a solute Rayleigh number of -10^8

Ta	R_{tcrit}
10	25 252 160.55
100	25 252 160.61
500	25 252 160.89
1000	25 252 161.23

the oscillatory mode. For $-283 < R_t < 605$, it is found that the system again becomes stable and for $R_t > 605$, instability sets in through the stationary mode. (In drawing the figure for the oscillatory mode, the values of R_t have been scaled down to accommodate the oscillatory and stationary curves in the same figure.) In conclusion, the results of the above linear analysis are:

(1) rotation, in general, inhibits the onset of the convective motion, though under certain conditions it destabilizes the system (Fig. 4);

(2) a rotating layer can get destabilized for a bottom heavy arrangement (Fig. 3);

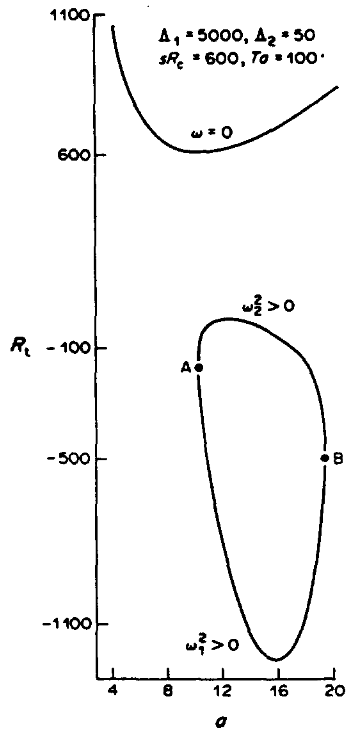
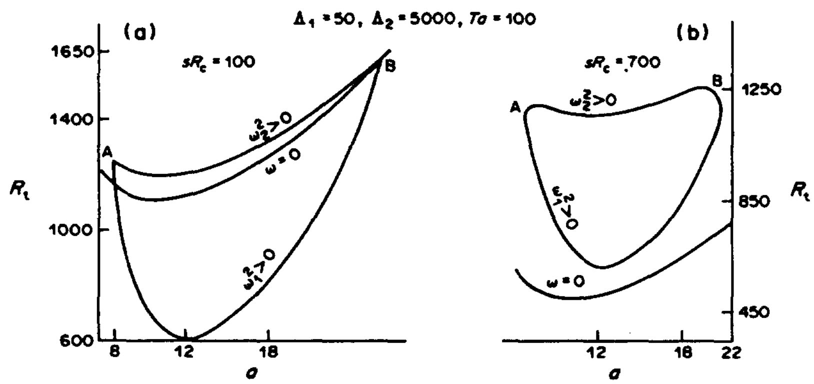


FIG. 6. Closed disconnected neutral stability curves showing the existence of three critical Rayleigh numbers. A, B are points of equal frequencies.

Table 3. Critical wave number a_c for various Darcy–Taylor numbers Ta and solute Rayleigh numbers R_c

	$\Lambda_1 = 4000$,	$\Lambda_2 = 400\,000$	$\Lambda_1 = 1000$,	$\Lambda_2 = 1500$
	$Ta = 500$	$Ta = 1000$	$Ta = 500$	$Ta = 1000$
sR_c	a_c	a_c	a_c	a_c
5000	14.8631 (S)	17.6709 (S)	14.8631 (S)	17.6709 (S)
4000	14.8631 (S)	17.6709 (S)	14.8631 (S)	50.0102 (O)
0	14.8631 (S)	17.6709 (S)	14.8631 (S)	47.5519 (O)
−4000	14.8752 (O)	17.7362 (O)	14.8631 (S)	45.2700 (O)
−5000	14.8679 (O)	17.7268 (O)	36.8482 (O)	44.8212 (O)



FIGS. 5(a), (b). Closed neutral stability curves showing the existence of two oscillatory modes. A, B are points of equal frequencies.

(3) critical wave number is independent of salinity Rayleigh number for stationary convection and for some data the change in the value of a_c is very slight for oscillatory convection (Table 1);

(4) for certain data rotation has a very little effect on the stability of the system (Table 2);

(5) for certain data there exist two positive frequencies which correspond to two oscillatory modes (Fig. 5);

(6) the linear stability criteria have to be expressed in terms of three thermal Rayleigh numbers as opposed to a single critical value for $\Lambda_1 > \Lambda_2$ (Fig. 6).

Most of the results observed in this analysis were also observed by Pearlstein [1] for a Newtonian fluid layer. While preparing the revised version of this paper the authors came across the article by Rudraiah *et al.* [14] which is for the Brinkman model.

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EFFET DE LA ROTATION SUR LA STABILITE D'UNE COUCHE FLUIDE DOUBLEMENT DIFFUSIVE DANS UN MILIEU POREUX

Résumé—On considère la stabilité convective d'un fluide doublement diffusif qui sature un milieu poreux tournant. On observe que (i) une couche peut être déstabilisée par un arrangement lourd en partie inférieure; (ii) en général, la rotation stabilise le système et le déstabilise sous certaines conditions; (iii) la rotation a un effet faible sur la stabilité de la couche fluide dans certaines conditions; (iv) parfois, il faut trois nombre de Rayleigh pour spécifier le critère de stabilité linéaire.

EINFLUSS EINER ROTATION AUF DIE STABILITÄT EINER DOPPELT DIFFUSIVEN FLUIDSCHICHT IN EINEM PORÖSEN MEDIUM

Zusammenfassung—Es wird die konvektive Stabilität eines doppelt diffusiven Fluids in einem rotierenden, porösen Medium betrachtet. Dabei ergibt sich: (i) eine rotierende Schicht kann dadurch destabilisiert werden, daß sie einen tiefliegenden Schwerpunkt aufweist; (ii) ganz allgemein stabilisiert eine Drehbewegung ein System, dies ist jedoch unter bestimmten Bedingungen nicht der Fall; (iii) die Drehbewegung hat unter bestimmten Bedingungen einen geringen Effekt auf die Stabilität der Fluidschicht; (iv) unter gewissen Bedingungen sind 3 Rayleigh-Zahlen als spezifische lineare Stabilitätskriterien erforderlich.

ВЛИЯНИЕ ВРАЩЕНИЯ НА УСТОЙЧИВОСТЬ СЛОЯ ЖИДКОСТИ С ДВОЙНОЙ ДИФФУЗИЕЙ В ПОРИСТОЙ СРЕДЕ

Аннотация—Исследуется конвективная устойчивость жидкости с двойной диффузией, насыщающей вращающуюся пористую среду. Найдено, что (i) устойчивость вращающегося слоя может быть нарушена в случае системы с массивным основанием; (ii) в целом вращение стабилизирует систему, хотя при определенных условиях может наблюдаться неустойчивость слоя; (iii) вращение может оказывать незначительное влияние на устойчивость жидкого слоя; (iv) при некоторых условиях для определения критериев линейной устойчивости необходимы три значения числа Рэлея.